Special Utilization of Triaxial Magnetometer Measurements as Prime Attitude Determination Sensors

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Mathematical and data analysis techniques are developed to determine the attitude of a rocket whose measuring devices encounter a partial failure. These devices consist of a set of triaxial magnetometers and a lunar (solar) aspect sensor. However, for this analysis, the lunar sensor provides only the times of its measurements, not its line-of-sight angles. Magnetometer measurements are corrected, and then used with the times of the lunar readouts and empirical lunar and magnetometer values to calculate lunar angle measurements. Standard attitude determination techniques are applied to find the elevation and azimuth of the rocket axis. Using the orientation of the rocket axis, attack angles with the lunar and magnetic field vectors are simulated. Further, if the rocket is well behaved and an orientation of its axis can be determined during this time period, then it is possible to calculate the axis of precession. These techniques are applied to rocket A09.209-1, whose primary attitude sensing devices consisted of a triaxial set of Schonstedt 600 Milligauss Magnetometers and a Bay Shore Lunar Sensor. The lunar sensor angular output was saturated for all but a few points on the lower altitude portion, but times of lunar measurements were available.

I. Introduction

THE inability to determine reliable attitude information when the attitude measuring systems encounter a partial failure has caused experimenters to disregard some of their experimental attitude-dependent measurements. This paper presents mathematical and data analysis techniques developed to calculate the angular values of a lunar or solar (hereafter referred to as lunar) aspect sensor which reads out only when the sensor, moon, and rocket axes are coplanar but only provides the times of the measurement. These angles are calculated using continuous corrected measurements of an onboard triaxial magnetometer system and times of the lunar measurement. Once these lunar angles are known, standard attitude determination techniques for a rocket whose attitude-sensing system consists of triaxial magnetometers and a lunar aspect sensor can be applied. For our standard attitude analysis, it is desirable to have simultaneous attitude measurements on a given vehicle axis.

If a rocket has a constant angular precessional velocity, then the axis of precession can be determined during this specified time interval given a single orientation of the axis, the precessional period, the half-cone angle, and acceptable magnetometer measurements. Having determined this axis, the generating equation for the rocket axis position, during the well-behaved period, can be expressed as a function of the axis of precession, the half-cone angle, and the vehicle angular velocity.

These techniques were applied to determine the attitude of the Ute-Tomahawk Rocket A09.209-1, fired at White Sands Missile Range, New Mexico. The attitude-measuring system of this rocket consisted of a set of triaxial Schonstedt 600 Milligauss Magnetometers and a Bay Shore System Lunar Sensor, model number LS-11-DR-2. The lunar sensor angular output was saturated for nearly the entire flight and only functioned for a few discrete points on the descent portion of the flight, but the times of the lunar readouts were determined. Two of the magnetometers which were perpendicular to the rocket axis were in the same plane as the lunar sensor but 45° out of phase. Therefore, for our attitude determination analysis, the lateral magnetometer measurements were phase shifted so that the sensing axis of one of them would have the same orientation as the lunar aspect sensor axis. Further, acceptable magnetometer measurements were difficult to achieve due to the effect on them of an onboard motor which had four magnets and created permanent magnetic fields which were modulated at the spin frequency of the motor. However, permanent and induced magnetic field corrections were estimated by a technique of solving a 3×3 system of linear equations using the actual flight data and the model Earth's magnetic field.

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II. Attitude Determination Procedures

Magnetometer Measurement Corrections

Vehicle attitude determination using an attitude-sensing combination of a lunar sensor and reliable magnetometer measurements has long been established.[1] The following discussion relates to corrected magnetometer measurements.

Since magnetometers sense the total ambient field, acceptable measurements require that all permanent, stray, and induced fields be accounted for as accurately as possible. Should preflight magnetometer calibrations be available, then permanent and induced magnetic field contributions can be determined. Frequently, however, preflight calibrations are not available, and the permanent and induced magnetic fields must be estimated using actual flight data.

To minimize the magnetometer output error for a triaxial orthogonal magnetometer system, we initialize the permanent field contribution $\alpha_i (i=1, 2, 3)$ for each of the magnetometers by data inspection. With the theoretical magnetic field M, we can form the equation

$$M^2 = A_1[\mathrm{MAG1}+\alpha_1]^2 + A_2[\mathrm{MAG2}+\alpha_2]^2 + A_3[\mathrm{MAG3}+\alpha_3]^2 \tag{1}$$

where MAG1, MAG2, MAG3 are the telemetered magnetometer outputs converted to milligauss by previously supplied vendor-calibration curves. The intermediate inducing factors $A_i\,(i=1,\,2,\,3)$ are solved by setting up a 3×3 linear system and using Gaussian elimination. A root mean square (rms) is then computed between the measured total magnetic field and the theoretical total field. The in-flight permanent field bias α_i for each magnetometer is updated within a specified iteration interval. Those values of α_i and A_i which cause the calculated field to converge within a $\pm 2\%$ tolerance limit of the total field and for which the rms is a minimum, are used as the permanent and induced field corrections to the magnetometer output.

Once acceptable magnetometer data are calculated, then along with the times of lunar measurements, it is possible to calculate lunar angular values.

Determination of Lunar Angular Values

Let $\hat{X}$, $\hat{Y}$, $\hat{Z}$ be an orthonormal system of unit vectors, with $\hat{X}$ parallel to the rocket axis, $\hat{Y}$ perpendicular to the rocket axis and in the direction of the lunar aspect sensor axis and one of the magnetometer axes, and $\hat{Z}=\hat{X}\times\hat{Y}$.

Since the lunar aspect sensor reads out only when the moon is in the plane of $\hat{X}$ and $\hat{Y}$, the unit moon vector $\hat{S}$ can be expressed in this vehicle system by

$$\hat{S}=\cos\beta_s\hat{X} + \sin\beta_s\hat{Y} \tag{2}$$

where β_s is the converted measured lunar angle.

The measured Earth's magnetic field vector can also be expressed in this vehicle system as

$$M=M_x\hat{X} + M_y\hat{Y} + M_z\hat{Z} \tag{3}$$

where M_x, M_y, M_z are the corrected measurements of the triaxial set of magnetometers.

Then expressing $\hat{M}$ as a unit vector

$$\hat{M}=\frac{M}{|M|}=\frac{M_x}{|M|}\hat{X}+\frac{M_y}{|M|}\hat{Y}+\frac{M_z}{|M|}\hat{Z}$$

$$=\cos\alpha_x\hat{X} + \cos\alpha_y\hat{Y} + \cos\alpha_z\hat{Z} \tag{4}$$

where $|M|=\sqrt{M_x^2+M_y^2+M_z^2}$

The scalar product of the $\hat{M}$ and $\hat{S}$ at the time of a lunar measurement is given by

$$(\hat{M}\cdot\hat{S})_1=\cos\alpha_x\cos\beta_s+\cos\alpha_y\sin\beta_s \tag{5}$$

The scalar product can also be derived from empirical sources[2] with the expressions

$$\hat{M}=\hat{e}_{\theta_c}x_m+\hat{e}_{\phi_c}y_m+\hat{e}_{r_c}z_m \tag{6}$$

and

$$\hat{S}=\hat{e}_{\theta_c}s_1+\hat{e}_{\phi_c}+\hat{e}_{r_c}s_3 \tag{7}$$

The system $\hat{e}_{\phi_c}$, $\hat{e}_{\theta_c}$, $\hat{e}_{r_c}$ is a local orthonormal system of unit vectors (Fig. 1) with $\hat{e}_{\theta_c}$ tangent to the meridan circle through the launcher and pointing to true North, $\hat{e}_{\phi_c}$ tangent to the circle of latitude through the launching tower and pointing East of North, and

$$\hat{e}_{r_c}=\hat{e}_{\phi_c}\times\hat{e}_{\theta_c}$$

Then

$$(\hat{M}\cdot\hat{S})_2=x_m s_1 + y_m s_2 + z_m s_3 \tag{8}$$

and with acceptable magnetometer measurements

$$(\hat{M}\cdot\hat{S})_1\approx(\hat{M}\cdot\hat{S})_2 \tag{9}$$

or

$$(\hat{M}\cdot\hat{S})_2=\cos\alpha_x\cos\beta_s + \cos\alpha_y\sin\beta_s \tag{10}$$

Dividing both sides by $\cos\beta_s$ and squaring them results in a quadratic equation involving the variable $\tan\beta_s$. Solving by conventional methods, we find

$$\tan\beta_s=\frac{-\cos\alpha_x\cos\alpha_y}{\cos\alpha_y^2-(\hat{M}\cdot\hat{S})_2^2}$$

$$\pm\frac{(\hat{M}\cdot\hat{S})_2\sqrt{\cos\alpha_x^2+\cos\alpha_y^2-(\hat{M}\cdot\hat{S})_2^2}}{\cos\alpha_y^2-(\hat{M}\cdot\hat{S})_2^2} \tag{11}$$

Since the angle between the moon and rocket axis at launch can be calculated and then updated with rate-of-change information available from the longitudinal magnetometer output, a tolerance limit can be established for β_s values and the ambiguity of sign in Eq. (11) resolved.

Determination of Elevation and Azimuth of Rocket at a Lunar Readout Time

The rocket axis $\hat{X}$ can be expressed as a linear combination of the three unit vectors, $\hat{M}$, $\hat{S}$, and $\hat{M}\times\hat{S}/|\hat{M}\times\hat{S}|$; i.e.

$$\hat{X}=\alpha\hat{M} + \beta\hat{S} + \gamma(\hat{M}\times\hat{S})/|\hat{M}\times\hat{S}| \tag{12}$$

Solving the linear system

$$\alpha=(\cos\alpha_x-(\hat{M}\cdot\hat{S})\ \cos\beta_s)/|\hat{M}\times\hat{S}|^2 \tag{13a}$$

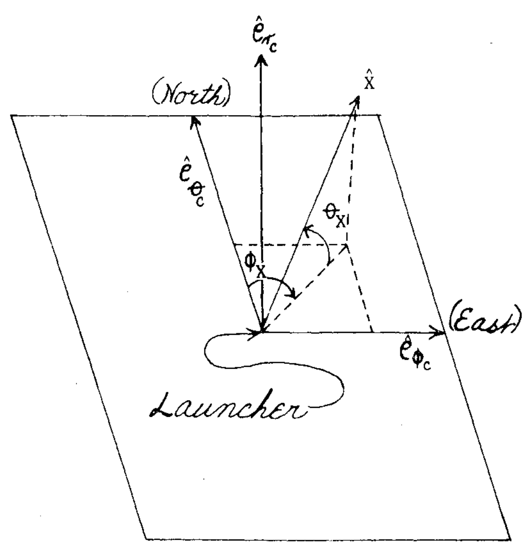
$$\beta=(\cos\beta_s-(\hat{M}\cdot\hat{S})\ \cos\alpha_x)/|\hat{M}\times\hat{S}|^2 \tag{13b}$$

$$\gamma=(\hat{M}\hat{S}\hat{X})/|\hat{M}\times\hat{S}|=-\cos\alpha_z\ \sin\beta_s/|\hat{M}\times\hat{S}| \tag{13c}$$

$\hat{X}$ can also be expressed in the $\hat{e}_{\phi_c}$, $\hat{e}_{\theta_c}$, $\hat{e}_{r_c}$ system as

$$\hat{X}=\hat{e}_{\theta_c}\cos\theta_x\cos\phi_x+\hat{e}_{\phi_c}\cos\theta_x\sin\phi_x+\hat{e}_{r_c}\sin\theta_x \tag{14}$$

where θ_x is the elevation of the rocket axis above the local horizontal plane at launch and ϕ_x is the azimuth of the rocket



θ_X = Elevation of $\hat{X}$

ϕ_X = Azimuth of $\hat{X}$

Fig. 1 Coordinate system.

axis measured positive East of North. Then

$$\sin\theta_x = \frac{\alpha z_m + \beta s_3 + \gamma(\hat{M}\hat{S}\ \hat{e}_{r_c})}{|\hat{M}\times\hat{S}|^2} \tag{15}$$

$$\tan\phi_x = \frac{\alpha y_m + \beta s_2 + \gamma(\hat{M}\hat{S}\ \hat{e}_{\phi_c})}{\alpha x_m + \beta s_1 + \gamma(\hat{M}\hat{S}\ \hat{e}_{\theta_c})} \tag{16}$$

Determination of the Axis of Precession

If the rocket axis has assumed a constant angular velocity of precession, then the axis of precession e_r can be expressed in the $\hat{e}_{\phi_c}$, $\hat{e}_{\theta_c}$, $\hat{e}_{r_c}$ system by

$$e_r = \hat{e}_{\theta_c}\cos\theta_q\cos\phi_p + \hat{e}_{\phi_c}\cos\theta_p\sin\phi_p + e_{r_c}\sin\theta_p \tag{17}$$

where θ_p and ϕ_p are the elevation and azimuth, respectively. e_r can also be expressed as a linear combination of three unit vectors $\hat{M}$, $\hat{X}$, and $\hat{M}\times\hat{X}/|\hat{M}\times\hat{X}|$. The angles θ_p and ϕ_p can be determined in a manner similar to that used for θ_x and ϕ_x, i.e.

$$e_r = \alpha_p\hat{M} + \beta_p\hat{X} + \gamma_p\frac{\hat{M}\times\hat{X}}{|\hat{M}\times\hat{X}|} \tag{18}$$

where

$$\alpha_p = (\cos\beta_h - \cos\alpha_x\cos\bar{\alpha})/\sin^2\alpha_x \tag{19a}$$

$$\beta_p = (\cos\bar{\alpha} - \cos\alpha_x\cos\beta_h)/\sin^2\alpha_x \tag{19b}$$

$$\gamma_p = (\hat{M}\hat{X}e_r)/|\hat{M}\times\hat{X}| \tag{19c}$$

and

$$\cos\bar{\alpha} = \text{half-cone angle} = \hat{X}\cdot e_r \tag{20}$$

$$\cos\beta_h = \hat{M}\cdot e_r \tag{21}$$

Then

$$\sin\theta_p = \frac{\alpha_p z_m + \beta_p\sin\theta_x + \gamma_p(\hat{M}\hat{X}e_{r_c})}{\sin^2\alpha_x} \tag{22}$$

$$\tan\phi_p = \frac{\alpha_p y_m + \beta_p\cos\theta_x\sin\phi_x + \gamma_p(\hat{M}\hat{X}\hat{e}_{\phi_c})}{\alpha_p x_m + \beta_p\cos\theta_x\cos\phi_x + \gamma_p(\hat{M}\hat{X}\hat{e}_{\theta_c})} \tag{23}$$

An ambiguity of sign that may occur in the expression for γ_p is resolved by generating β_s values using both possible solutions. These two solutions are compared with the previously calculated β_s values. The solution that compares best in phase and structure determines the sign to be used.

Generation of Attitude Data

Once the axis of precession is determined, then the generating equation for the rocket axis $\hat{X}$ at any time t, as described in Ref. 3, is

$$\hat{X}(t) = e_r\cos\bar{\alpha} + e_\theta\sin\bar{\alpha}\cos(\omega_0 t + \chi) + e_\phi\sin\bar{\alpha}\sin(\omega_0 t + \chi) \tag{24}$$

where

$$e_\theta = \partial e_r/\partial\theta_p = -e_{\theta_c}\sin\theta_p\cos\phi_p - \hat{e}_{\phi_c}\sin\theta_p\sin\phi_p + \hat{e}_{r_c}\cos\theta_p \tag{25}$$

$$e_\phi = 1/\cos\theta_p\frac{\partial e_r}{\partial\phi_p} = -\hat{e}_{\theta_c}\sin\phi_p + \hat{e}_{\phi_c}\cos\phi_p \tag{26}$$

$$\chi = \tan^{-1}\{(\hat{M}\cdot e_\phi)/(\hat{M}\cdot e_\theta)\} - \omega_0 t_0$$

at a time t_0 of the maximum of the axial magnetometer.

Transforming $\hat{X}(t)$ to the $\hat{e}_{\phi_c}$, $\hat{e}_{\theta_c}$, $\hat{e}_{r_c}$ system and expressing in terms of direction cosines

$$\hat{X}(t) = \hat{e}_{\theta_c}\cos\alpha_\theta + \hat{e}_{\phi_c}\cos\alpha_\phi + \hat{e}_{r_c}\cos\alpha_r \tag{27}$$

where

$$\cos\alpha_\theta = \hat{X}(t)\cdot\hat{e}_{\theta_c} \tag{28a}$$

$$\cos\alpha_\phi = \hat{X}(t)\cdot\hat{e}_{\phi_c} \tag{28b}$$

$$\cos\alpha_r = \hat{X}(t)\cdot\hat{e}_{r_c} \tag{28c}$$

$\hat{X}(t)$ can also be expressed in terms of its elevation θ_x and azimuth ϕ_x angles by

$$\hat{X}(t) = \hat{e}_{\theta_c}\cos\theta_x\cos\phi_x + \hat{e}_{\phi_c}\cos\theta_x\sin\phi_x + \hat{e}_{r_c}\sin\theta_x \tag{29}$$

Then by equating coefficients for both expressions of $\hat{X}(t)$, the attitude at any time t is given by

$$\theta_x = \sin^{-1}(\cos\alpha_r) \tag{30}$$

$$\phi_x = \tan^{-1}(\cos\alpha_\phi/\cos\beta_\theta) \tag{31}$$

Generating Magnetometer and Lunar Measurements

The simulation of the angles that the rocket axis makes with the moon and the Earth's magnetic field was performed by taking scalar products of $\hat{S}$ and $\hat{M}$ with $\hat{X}(t)$

$$\beta_s(\text{generated}) = \cos^{-1}(\hat{S}\cdot\hat{X}(t))$$
$$= \cos^{-1}(s_1\cos\theta_x\cos\phi_x + s_2\cos\theta_x\sin\phi_x + s_3\sin\theta_x) \tag{32}$$

$$\alpha_x(\text{generated}) = \cos^{-1}(\hat{M}\cdot\hat{X}(t))$$
$$= \cos^{-1}(x_m\cos\theta_x\cos\phi_x + y_m\cos\theta_x\sin\phi_x + z_m\sin\theta_x) \tag{33}$$

III. Rocket A09.209-1

This analysis was applied to the rocket A09.209-1, whose primary attitude sensing devices on board consisted of a triaxial set of magnetometers and a lunar sensor. Also aboard was a set of photometers which provided approximate lunar information. With these sensors, it was possible to calculate the attitude for onboard probes at any time during the flight.

Lunar Sensor

The Bay Shore lunar sensor had its sensitivity set to read out a wide range of high lunar phase angles with a resolution of 2° and an accuracy of ±1°. The sensor was mounted in such a way that it read out only when the rocket axis, the moon, and the sensor axis were all in the same plane (once per spin of the rocket). The sensor output, which was in the form of a digital code, could be converted directly to lunar aspect angles. The angles this sensor was capable of reading ranged from +70° to −26° as measured from the centerline of the sensor.

Photometers

On board this vehicle was a set of three solar blind photomultipliers with uv interference filters with a limited field of view. This set of photometers was mounted in such a manner that the projections of the three photometers on the plane perpendicular to the rocket axis were coincident with projection of the lunar sensor, i.e., the azimuth angles in the horizontal plane were equal. However, only one of the photometers was aligned with the lunar sensor and this photometer failed to function properly. The others, one with a 2750 angstrom filter which was mounted up 5° from the horizontal plane and the other with 2600 angstrom filter which was mounted up 10° from the horizontal plane, functioned properly.

The output from these two photometers provided some lunar information since they indicated maximum intensities when sensing the moon.

Magnetometers

The magnetometer system on board was a triaxial fluxgate magnetometer, specifically designed for use in rocket attitude measurement systems. A measured field component within the range of ± 600 milligauss was converted to an analog voltage defined by the equation

$$E = 2.40 + .004\,H\cos\phi$$

where E is the output in volts, H is the ambient field in milligauss, and ϕ is the angle between the magnetic field vector and sensor's positive magnetic axis. The vendor's calibrations for this system were not performed under simulated flight conditions.

Errors in Magnetometer Readouts

On this flight was a Siemens brushless dc motor with four permanent magnets which spun at approximately 3000 rpm. This motor was located 30° off the longitudinal axis and in a plane parallel to door 1 (Fig. 2) and modulated the voltage output of the three magnetometers. The absence of simulated flight condition calibrations, coupled with the effects of the motor on the magnetometers, made normal, permanent, and induced field corrections extremely difficult. The effect of the motor on all three magnetometers is demonstrated in Fig. 3a-c, where

$$\mathrm{MAG1} \simeq .112 \text{ volts} \simeq 26.88 \text{ milligauss}$$

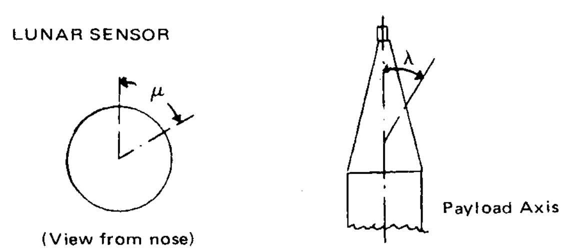
$$\mathrm{MAG2} \simeq .147 \text{ volts} \simeq 35.28 \text{ milligauss}$$

$$\mathrm{MAG3} \simeq .3 \text{ volts} \simeq 72 \text{ milligauss}$$

The effect of this motor on the magnetometers was removed from the measurements by using averaging techniques,[4] and best estimates[5] of the permanent and induced fields on each magnetometer were determined by Eq. (1).

Magnetometer Phase Shifting

To properly utilize the standard attitude determination technique mentioned, it is desirable to have simultaneous



INSTRUMENT	μ (degrees)	λ (degrees)
MAG1	0	0
MAG2	225°	90°
MAG3	315°	90°
Lunar Sensor	0	90°
2750 A Photometer	0	85°
2600 A Photometer	0	80°
Siemens Brushless Motor	90°	30°

Fig. 2 Ute-Tomahawk A09.209-1 instrument orientations.

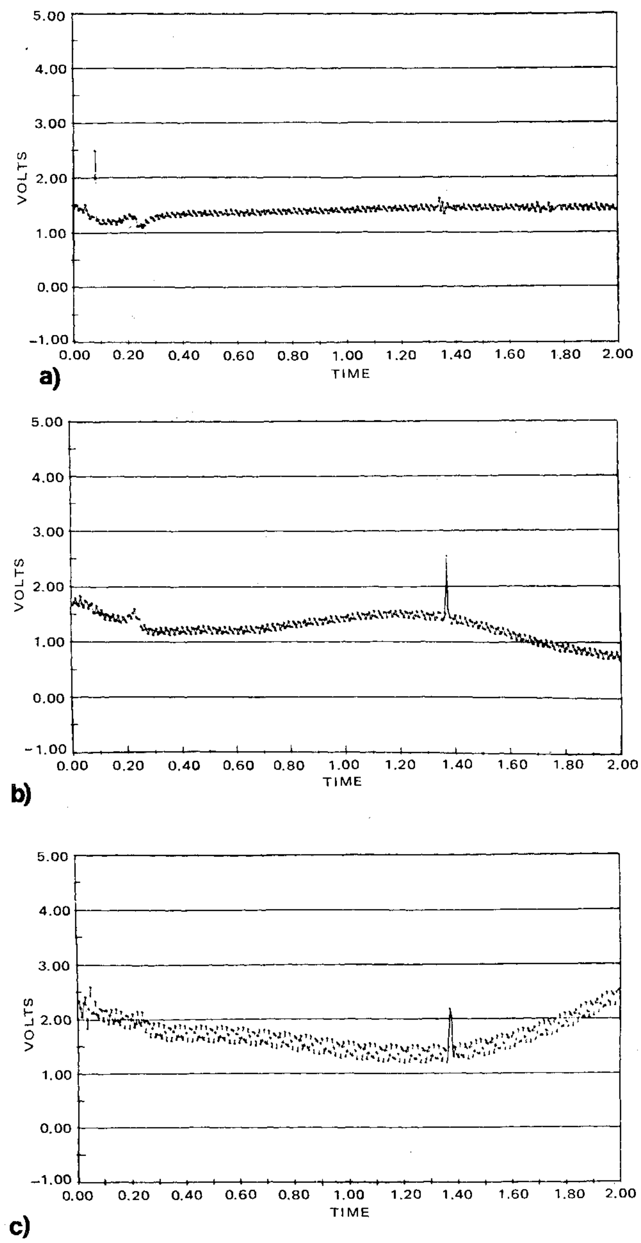


Fig. 3 The effects of the motor on a) MAG1, b) MAG2, and c) MAG3.

readings of the Earth's magnetic field and the moon from attitude-sensing devices with the same orientation on the vehicle. On this particular vehicle, the lunar sensor and both lateral magnetometers were mounted normal to the axis of the rocket, but MAG3 and the lunar sensor were out of phase by 45° (Fig. 2).

To properly align a magnetometer axis parallel to the lunar sensor axis, we begin by defining the unit vectors $\hat{U}$ and $\hat{V}$ along the U and V magnetometer axes, respectively, as

$$\hat{U} = \hat{e}_{\theta_c}\cos\theta_u\cos\phi_u + \hat{e}_{\phi_c}\cos\theta_u\sin\phi_u + \hat{e}_{r_c}\sin\theta_u \tag{34}$$

$$\hat{V} = \hat{e}_{\theta_c}\cos\theta_v\cos\phi_v + e_{\phi_c}\cos\theta_v\sin\phi_v + \hat{e}_{r_c}\sin\theta_v \tag{35}$$

Then the unit vector $\hat{Y}$ along the lunar sensor axis and in the plane of $\hat{U}$ and $\hat{V}$ can be expressed as

$$\hat{Y} = \hat{U}\cos\Omega + \hat{V}\sin\Omega \tag{36}$$

where Ω is the angle between the lunar sensor axis and $\hat{U}$ magnetometer axis.

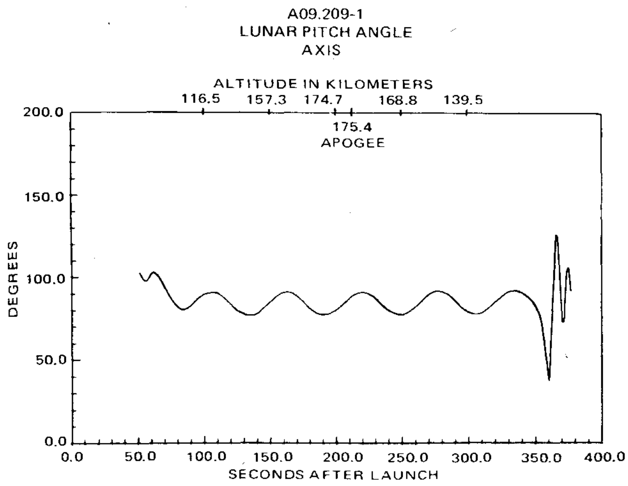


Fig. 4 A09.209-1 lunar pitch angle of the vehicle axis.

The scalar product of the Earth's unit magnetic field vector $\hat{M}$ as defined in Eq. (6) with $\hat{Y}$ is given by

$$\cos\alpha_y = (\hat{M}\cdot\hat{Y}) = (\hat{M}\cdot\hat{U})\cos\Omega + (\hat{M}\cdot\hat{V})\sin\Omega \tag{37}$$

where $(\hat{M}\cdot\hat{U})$ and $(\hat{M}\cdot\hat{V})$ are equal to the actual normalized magnetometer outputs M_u and M_v. Then

$$\cos\alpha_y = M_u\cos\Omega + M_v\sin\Omega \tag{38}$$

is the expression that represents the cosine of the angle between the magnetic field and the lunar sensor axis.

A09.209-1 Attitude Results

Continuous attitude information was required for the entire time the photometers functioned properly, i.e., 51-376 sec after launch. This time interval included the lower altitude portions of the flight during which the rocket's axis of precession was not constant and the vehicle not well behaved.

For the ascent portion of the flight from 51-91.2 sec, no lunar angle information was available. However, lunar angular values were calculated using the corrected magnetometer output, its behavioral pattern, and the times of lunar readouts. With this lunar information and magnetometer output, we were then able to determine attitude of the rocket axis.

During the well-behaved portion from 91.2-348 sec, analysis of the magnetometer measurements indicated the vehicle to have a constant angular velocity of precession. Attitude and β_s values during this interval were therefore calculated using procedures described in Sec. II. The calculated data compared favorably with the behavioral characteristics of the photometer data.

During descent, two types of lunar information were available for attitude determination. The first consisted of lunar sensor angular outputs at a few discrete points, but these angles were reflecting a 4° cone of uncertainty. The equations for elevation and azimuth of the rocket axis discussed in Sec. II were used to calculate the attitude for these points. The second type was provided by corrected magnetometer measurements and only the times of lunar readouts. This information, together with the discrete attitude points, provided sufficient measurements with which to calculate attitude during this lower altitude portion of flight. The calculated angle between the lunar probe and moon vector fell within the 4° cone of uncertainty established by the discrete lunar angle measurements and also agreed with the behavioral structure of the photometer measurements.

The estimated lunar attack angle with the vehicle axis is displayed in Fig. 4.

IV. Concluding Remarks

By referencing appropriate vectors to the fixed inertial system, the attitude determination procedures discussed in this paper can also be applied to satellites. Onboard information is defined as in Eqs. (2) and (4). If necessary, a modification to the vector $\hat{S}$ can easily be made to accommodate a contribution in the $\hat{Z}$ direction. When considering offboard information, the rotation of the Earth about its axis and its rotation about the sun must be taken into account. To express attitude in terms of a geocentric system of coordinates would not suffice due to the above-mentioned angular contributions. A convenient fixed system $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ can be defined with respect to the Vernal Equinox with the unit vector $\boldsymbol{i}$ pointing from the Vernal Equinox toward the sun, the unit vector $\boldsymbol{k}$ parallel to the north polar axis, and $\boldsymbol{j} = \boldsymbol{k} \times \boldsymbol{i}$. The unit vector $\boldsymbol{j}$ lies in the equatorial plane of the Earth. In this reference system, direction cosines can be derived from Eqs. (6) and (7).

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